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A method of tracking ensembles of particle images

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Abstract A method for tracking ensembles of particle images of a digital PIV record by making use of the "minimum quadratic difference" (MQD) technique is described. The similarity of this method with correlation schemes used for the evaluation of PIV is discussed. The method is exemplified with two applications that are also evaluated by auto- and cross-correlation, respectively.

1 Introduction

A number of methods and algorithms for the evaluation of particle-image-velocimetry (PIV) records have been reported; see, e.g., Adrian (1991). In order to apply one of these evaluation procedures the PIV record must be available in digitized form. Particle image patterns obtained with double pulsed illumination systems often are evaluated with an autocorrelation algorithm (Keane and Adrian 1990; Heckmann et al. 1994). If single exposures of the particle image patterns are recorded with a defined time interval between exposures, evaluation by means of cross-correlation algorithms becomes possible (Willert and Gharib 1991); this has a number of advantages in comparison to the autocorrelation, particularly the removal of the directional ambiguity of the velocity vectors. Refinements of the evaluation by cross-correlation by applying a digital filter technique were reported, e.g., by Kemmerich and Rath (1994), while Huang et al. (1993a, b) describe a method for determining the displacement of ensembles of particle images ("particle image patterns" = PIP) by means of a cross-correlation, thereby achieving a higher degree of accuracy in the case of deformations of the patterns in the flow.

In this note we report on a method for the evaluation of PIV records that is different from the mentioned correlation schemes. The method is based on the principle of minimizing the (quadratic) differences between multi-component vectors or matrices in order to investigate the degree of similarity existing between such quantities. This principle is frequently used as a tool for analyzing or minimizing errors for numerical approximations or mathematical statistics. In the following we shall describe this "minimum quadratic difference" (MQD) method which, in principle, is a tracking algorithm and then present two applications including a comparison with existing correlation methods.

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2 Minimum quadratic difference method

2.1 Description of the method

The difference of the two matrices

$$G(i,j)=\begin{Bmatrix} g_{11} & g_{21} & \cdots & g_{M1} \\ g_{12} & g_{22} & \cdots & g_{M2} \\ \vdots & & & \vdots \\ g_{1N} & g_{2N} & \cdots & g_{MN} \end{Bmatrix} \quad \text{and}$$

$$G'(i,j)=\begin{Bmatrix} g'_{11} & g'_{21} & \cdots & g'_{M1} \\ g'_{12} & g'_{22} & \cdots & g'_{M2} \\ \vdots & & & \vdots \\ g'_{1N} & g'_{2N} & \cdots & g'_{MN} \end{Bmatrix}$$

each having $(M \cdot N)$ elements $g_{i,j}$ and $g'_{i,j}$, respectively, is given by

$$|G-G'| = [(g_{11}-g'_{11})^2+(g_{21}-g'_{21})^2+\cdots+(g_{M1}-g'_{M1})^2 + (g_{12}-g'_{12})^2+\cdots+(g_{MN}-g'_{MN})^2]^{1/2}$$

or

$$|G-G'| = \sqrt{\sum_{i=1}^{M}\sum_{j=1}^{N}(g_{ij}-g'_{ij})^2} \tag{1}$$

g_{ij}, g'_{ij} are considered here to be the gray values of the pixels (i, j) in limited areas ("patterns") of size $(M \cdot N)$ pixels of a digitized PIV record. The matrices G, G' describe certain distributions of limited numbers of digital particle images. The two patterns G and G' are the result of two separate records of particle image fields with a time interval Δt between the two records. The areas covered by G and G' are only small fractions of the whole particle image field. The object is to track the pattern displaced by an (average) vector (m^*, n^*) during the time interval Δt. The displacement (m^*, n^*) can be found by determining the minimum of the quadratic difference

$$D(m,n)=\frac{1}{M \cdot N}\sum_{i=1}^{M}\sum_{j=1}^{N}(g(i,j)-g'(i+m,j+n))^2 \tag{2}$$

in varying the values of m and n. The numbers i, j in Eq. (2) refer to one common coordinate system in the plane in which the two patterns G, G' are situated, in contrast to Eq. (1) where the numbers i, j apply independently to each pattern G, G'. A maximum of similarity of the patterns G and G' is established for the minimum of D. The equality $G(i, j) = G'(i - m^*, j - n^*)$ holds only for a solid displacement by (m^*, n^*) and identical imaging and illumination conditions of the two records yielding G and G'. The factor $(M \cdot N)^{-1}$ in Eq. (2) facilitates the use of patterns of different size during the evaluation of a PIV record.

In the application to a PIV double exposure, one has the superposition of two particle image patterns, and the difference (2) is found to have a central minimum $D = 0$ for $(m, n) = (0, 0)$. Two minima with $D_{min} > 0$ appear almost symmetrically on both sides of the central minimum (Fig. 1a), and the displacement to be determined follows from the positions (m^*, n^*), $(-m^*, -n^*)$ of these second order minima. This form of $D(m, n)$ is similar to the case, when the PIV double exposure is evaluated by means of an autocorrelation algorithm, and like there, a directional ambiguity is involved in the evaluation.

Such a similarity with existing evaluation schemes is also evident when the MQD method is applied to the evaluation of two successive PIV single exposures: there is no central minimum; only one minimum indicates the displacement (m^*, n^*), i.e., the directional ambiguity is removed (Fig. 1b), like in the application of the cross-correlation function.

2.2
Comparison with correlation schemes

In order to compare the described MQD method with evaluation schemes using correlation functions we set in Eq. (2) $g'(i, j) = g'(i + m, j + n)$ and write the equation as

$$\frac{M \cdot N}{2} D(m, n) = \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{N} g(i, j)^2 - \sum_{i=1}^{M} \sum_{j=1}^{N} g(i, j) g'(i + m, j + n) + \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{N} g'(i + m, j + n)^2 \quad (3)$$

The second term on the right side is the (negative) correlation function $R(m, n)$. The first term is a constant, while the third term also varies with (m, n). The position of the minimum of $D(m, n)$ coincides with the position of the maximum of $R(m, n)$ (second term) only, if the third term can be taken as a constant, i.e., independent of (m, n). This term is the summation of the quadratic gray values of all pixels included in the pattern $G'(i + m, j + n)$, and the dependence on (m, n) may be negligible when the particle images are uniformly distributed over the whole PIV record and equally illuminated.

However, this comparison is based only on formal aspects, and it should be clear that the principle of an evaluation by correlation functions (auto and cross-correlation) is different from the present MQD method that uses a tracking principle. The accuracy of the particle tracking is controlled by the MQD technique. It should also be considered that the area ("pattern") to which $R(m, n)$ from Eq. (3) applies is different from the area ("window") evaluated by utilizing correlation functions in "traditional" evaluation schemes.

3
Examples of application

We present two examples of applying the MQD tracking method to cases that are also evaluated with one of the "traditional" methods, auto and cross-correlation, respectively. Figure 2a is the digitized record taken with the air flow in a straight circular pipe at a mean velocity of approximately 15 m/s and a Reynolds number $Re_D = 1 \times 10^5$ (from Schlüter 1995). The velocity profile is not fully developed and it exhibits deviations from pure axial flow due to upstream disturbances. The record was evaluated with an auto-correlation algorithm, using two different sizes of the interrogation window, 50×50 and 80×80 pixels, respectively. In scanning the PIV record the interrogation window was moved in steps of 10 pixels. Results of the evaluation by autocorrelation are shown in Fig. 2b, c in form of velocity vectors. The use of the smaller window results in a large number of obviously wrong vectors. The larger size of the 80×80 pixel window and the high degree of overlap cause an intensive smoothing of the results, and less erroneous vectors appear in this case. But with the larger window the

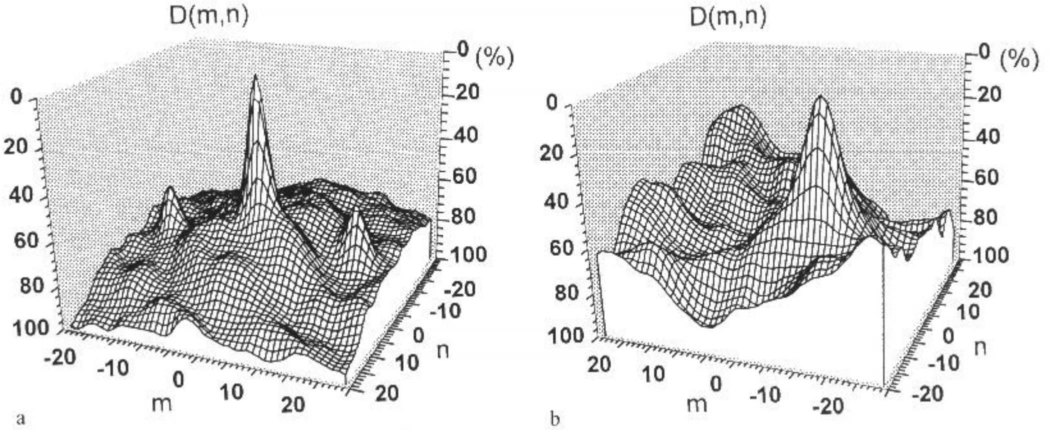

Fig. 1a, b. Difference $D(m, n)$ according to Eq. (3) applied to **a** PIV double exposure: displacement (m^*, n^*) of the tracked "pattern" is (17, 0) or (−17, 0) (in pixels), **b** two consecutive PIV single exposures: displacement (m^*, n^*) is found from the position of the (first order) minimum as (−10, −10) (in pixels)

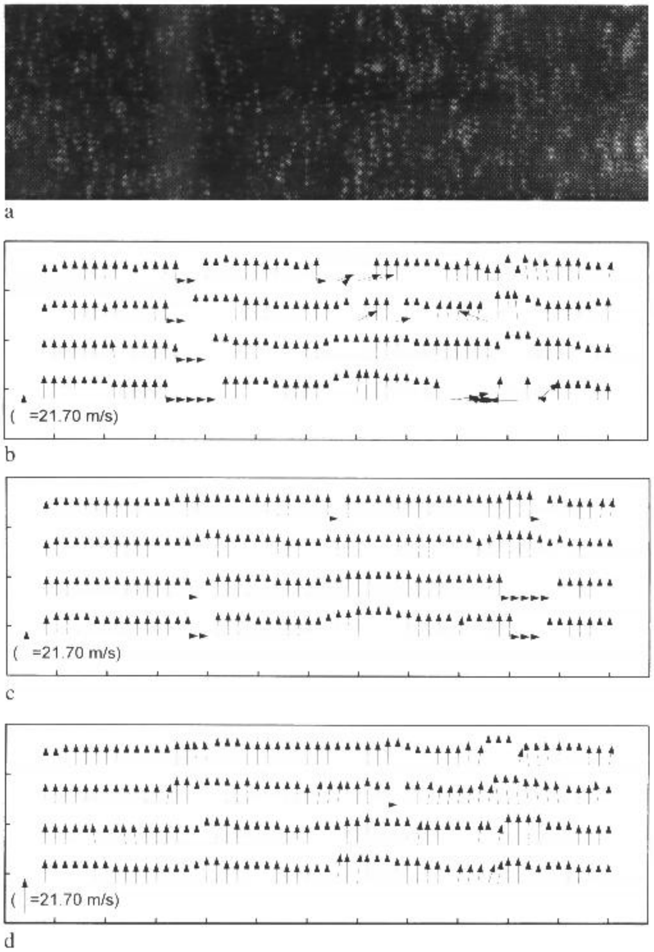

Fig. 2. **a** PIV double exposure of flow in a straight pipe after digitization. The 640×200 pixels PIV record covers an area of $83 \times 26\ mm^2$ in the light sheet, **b** result of evaluation by autocorrelation with 50×50 pixels interrogation window, **c** result of evaluation by autocorrelation with 80×80 pixels interrogation window, **d** result of evaluation by the MQD tracking method using a 50×50 pixels "pattern"

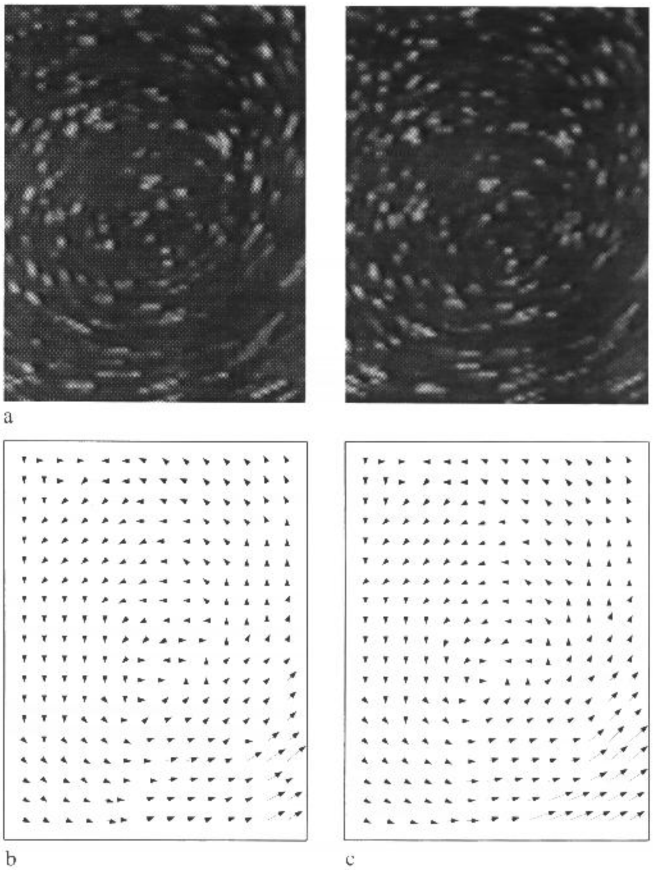

Fig. 3. **a** Two consecutive PIV single exposures of a recirculating water flow; time interval between exposures $\Delta t = 20$ ms. The 150×200 pixels records cover an area of $29 \times 39\ mm^2$ in the light sheet, **b** result of evaluation by cross-correlation using a 35×35 pixels window. The maximum velocity is 8.8 mm/s, **c** result of evaluating by the MQD tracking method using a 35×35 pixels "pattern"

velocity is averaged over a higher number of particle images, and the local resolution of the measurement decreases drastically. For the tracking with the MQD method "patterns" of size 50×50 pixels ($M \times N$ according to Eq. (3)) were used (Fig. 2d). In comparison to the 50×50 pixels autocorrelation window the results appear much more reliable, in the case of Fig. 2d with only one obviously wrong vector, and with a reasonable local resolution, as indicated by the variation in the lengths of the velocity vectors.

A comparison of the MQD tracking with an evaluation by cross-correlation is presented in Fig. 3. The particle image records shown (Fig. 3a) are the two successive half fields of a video frame separated by a time interval $\Delta t = 20$ ms (Xiong et al. 1996). A recirculating flow regime in the water flow through a transparent model reactor tank has been investigated.

The interrogation window used for the evaluation by cross-correlation has a size of 35×35 pixels, and the "pattern" tracked with the MQD method is of the same size. In comparing the results obtained with the two methods (Fig. 3b, c) one recognizes that the number of erroneous vectors is considerably lower for the MQD method, and that details of the flow are better represented in this case.

4
Conclusion

The described MQD method for the evaluation of PIV records is a tracking algorithm. In contrast to "traditional" tracking schemes (e.g., Grant and Smith 1988) no single particles but ensembles of particle images, that we call here "patterns", are tracked. This allows the application of the tracking principle to much higher particle number densities than single-particle-tracking methods. The size of the window to be tracked is comparable to the usual size of the interrogation windows used with correlation techniques, or it is even smaller, then providing a higher local resolution than those techniques. A disadvantage in comparison to the correlation methods might be that, until now, it appears that the FFT technique cannot be applied for fast processing.

This MQD tracking method can be applied to both PIV double exposures and consecutive single exposures that are evaluated traditionally by auto- and cross-correlation, respectively. The obvious similarity to the correlation schemes is only formal. In the two examples shown the evaluation by MQD tracking is apparently more accurate than with auto- or cross-correlation. Whether this is a general issue has to be found out in larger numbers of additional checks and comparisons that are presently under study.

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