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Generating arbitrarily sized interrogation windows for correlation-based analysis of particle image velocimetry recordings

L. Gui, W. Merzkirch

Abstract We describe a technique that allows an arbitrary size of the interrogation window when using the traditional FFT algorithm in analysing PIV recordings by either cross- or auto-correlation methods. The length and width of the effective interrogation window are no longer required to be composed of a number of pixels making a power of 2 (16, 32, 64 etc). This gives a higher flexibility in selecting the appropriate window size.

1 Introduction

Evaluation of digital particle image recordings by either auto-correlation or cross-correlation methods has become a standard procedure see, e.g., Adrian (1991), Willert and Gharib (1991), Heckmann et al. (1994). In order to accelerate the computation of the correlation function of the particle image patterns in the interrogation windows and for searching the maxima of this function one makes use of the FFT algorithm. The application of the FFT requires the number of pixels M, N, expressing the width and height of the interrogation windows, to be powers of 2, e.g. 16, 32, 64, etc. This is a strong limitation in choosing the optimum window size. A compromise between two contradictory criteria has to be found in selecting the appropriate window size: A large window allows the presence of a sufficiently high number of particle images as it is needed for an accurate determination of the positions of the maxima of the correlation function (Keane and Adrian 1990). A small window with fewer particle images reduces the influence of an averaging in determining the velocity vector and thereby increases the local resolution of the velocity measurement.

In this note we describe a simple technique that, in combination with applying the FFT algorithm, allows the choice of an interrogation window of arbitrary size, i.e. the number of pixels of each window side is no longer required to be a power of 2. The technique might be of practical interest due to its simplicity in comparison to algorithms based on the "mixed radix FFT" (Singleton 1969) and providing a variety of sets of fast transforms for window sizes $M, N \neq 2^k$. It should be mentioned that an alternative is the direct computation of the cross-correlation in the image space using suitable speed enhancement (Roesgen and Totaro 1995).

2 Derivation of the technique

The PIV recording is a distribution of grey values $G(x, y)$ in the x-y plane. We assume that G is available in digital form for each pixel (i, j) in this plane. The velocity vector (u, v) is determined at a position (x_m, y_n) by evaluating the content (grey values) $g(i, j)$ of an interrogation window of size $M \times N$ pixels (Fig. 1a). The relationship between G and g is

$$g(i,j)=G\left(x_m-\frac{M}{2}+i,\, y_n-\frac{N}{2}+j\right) \quad \text{with}$$
$$0 \leqslant i \leqslant M-1,\, 0 \leqslant j \leqslant N-1 \tag{1}$$

For two separate recordings G_1, G_2, the cross-correlation function to be determined is (Willert and Gharib 1991)

$$\Phi(m,n)=\frac{\sum_{i=-\infty}^{+\infty}\sum_{j=-\infty}^{+\infty} g_1(i,j)\,g_2(i+m,\ j+n)}{\sum_{i=-\infty}^{+\infty}\sum_{j=-\infty}^{+\infty} g_1(i,j)\ \sum_{i=-\infty}^{+\infty}\sum_{j=-\infty}^{+\infty} g_2(i,j)} \tag{2}$$

In Eq. (2) the samples g_1, g_2 are defined in the infinite plane $(-\infty < i < +\infty,\ -\infty < j < +\infty)$. In practice, however, g_1, g_2 (or g) are restricted to the finite plane of the interrogation window. The application of the FFT for determining Φ in the window then requires to assume g as being distributed periodically in the i–j plane with the periodicity M, N (Fig. 1b), i.e.

$$g(i,j)=g(i+kM,\, j+lN), \quad k, l=0,\ \pm1,\ \pm2,\ \pm3,\ \ldots \tag{3}$$

The discussion applies equally to cross-correlation and auto-correlation algorithms, and we only refer to g instead of distinguishing between g_1 and g_2.

The correlation calculation with the FFT can be performed even if the periodically arranged PIV samples are framed by equally-spaced, parallel stripes of uniform grey value. For, these stripes do not contribute new information relevant for the velocity field. If we add the frame to the window, then the total window consists of the interrogation area (PIV sample) and the adjacent uniform part of the frame. While the total

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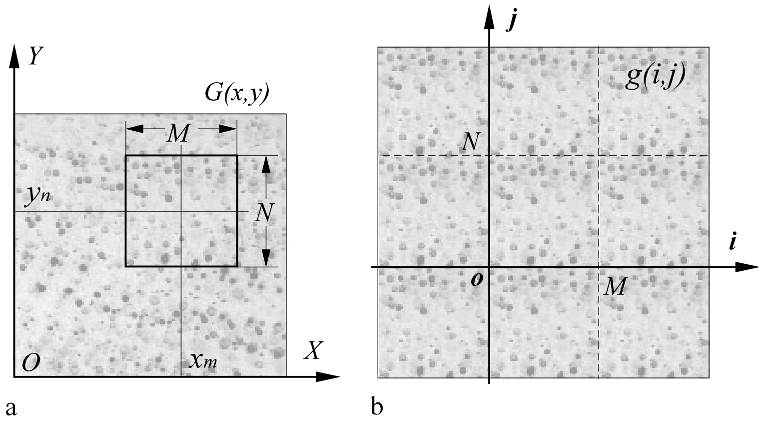

Fig. 1. **a** Position of the interrogation window in the PIV recording having a distribution of grey values $G(x, y)$; **b** periodical repetition of the sample (window) $g(i, j)$ in the i–j plane

window must still be composed of a number of pixels ($M^* \times N^*$) making powers of 2, e.g. 64×64 pixels, we can now allow the interrogation area to be of arbitrary size ($M \times N$), in our example less than 64×64 but more than 32×32 pixels as shown in Fig. 2.

The correlation function applying to the total window is now (again, we only refer to the case of cross-correlation and omit the constant denominator of Eq. (2)):

$$\Phi^*(m, n) = \sum_{i=0}^{M^*-1} \sum_{j=0}^{N^*-1} g_1^*(i, j) g_2^*(i+m, j+n)$$

$$\text{with } \begin{cases} M^* = 2^\alpha & \text{if } 2^{\alpha-1} < M \leqslant 2^\alpha \\ N^* = 2^\beta & \text{if } 2^{\beta-1} < N \leqslant 2^\beta \end{cases} \alpha, \beta \text{ integers.} \tag{4}$$

The symbol ($*$) here refers to the total window. The function to be correlated, g^*, is related to the original pattern G of the PIV recording by

$$g^*(i, j) = G\left(x_m - \frac{M}{2} + i, y_m - \frac{N}{2} + j\right), \quad 0 \leqslant i \leqslant M-1, \; 0 \leqslant j \leqslant N-1 \tag{5}$$

in the interrogation part of the window. To the "frame" we assign the mean grey value $\bar{g}$ of the respective interrogation sample, so that g^* is described here by

$$g^*(i, j) = \bar{g} = \frac{1}{MN} \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} g^*(p, q).$$

with $M \leqslant i \leqslant M^* - 1$, $0 \leqslant j \leqslant N^* - 1$ and $0 \leqslant i \leqslant M^* - 1$, $N \leqslant j \leqslant N^* - 1$

For the FFT calculations g^* has to be taken periodical with the wave lengths M^*, N^*.

It remains to be shown that the correlation of the window including the "frame" yields the same result as correlating solely the interrogation area; this will be done in the following section by means of an example.

As a consequence of the inclusion of the artificial frame area in the evaluation window, the maximum displacement that can be measured is allowed to be more than half of the effective diameter of the $M \times N$ interrogation area (Nyquist theorem) or, if one follows Willert and Gharib (1991), more than one third of that diameter. This apparent increase of the dynamic range of the measurement is the result of the larger data sets transformed by the FFT.

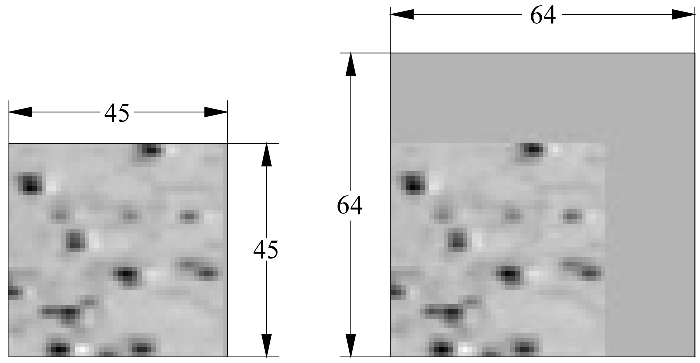

Fig. 2. Effective interrogation area of size 45×45 pixels (left) imbedded in the total window of 64×64 pixels that includes the "frame" of constant grey value $\bar{g}$ (right)

3 Examples

We first test the influence of the "frame" on the accuracy of the evaluation. A digital single exposure taken with tracer particles in a water flow serves as the reference object, and two successive PIV recordings that have to be analyzed are artificially produced by moving the reference exposure in definite steps of an integer number of pixels. In this way we have a realistic distribution of particle images, and a controlled constant particle image displacement. The aim is to determine the accuracy of the evaluation as a function of the magnitude of the displacement.

Evaluation by cross-correlation is performed with an interrogation window of size 32×32 pixels. The diameter of the particle images is between 3 and 6 pixels, 6 to 10 particle images are present within the window area. First, the recording is analyzed in the conventional way, i.e. without "frame"; then, a frame of the same width as the window is used in performing the evaluation with FFT (Fig. 3a). The result obtained for each interrogation window will deviate to a certain degree from the true displacement due to imperfections in the correlation analysis. The rms value of the derivations for all window positions applied to an artificial PIV recording, i.e. a test case with given constant displacement, is shown in Fig. 3b as a function of the displacement (number of pixels). It is seen that the accuracy for the evaluations without and with "frame" is about the same, and the figure indicates

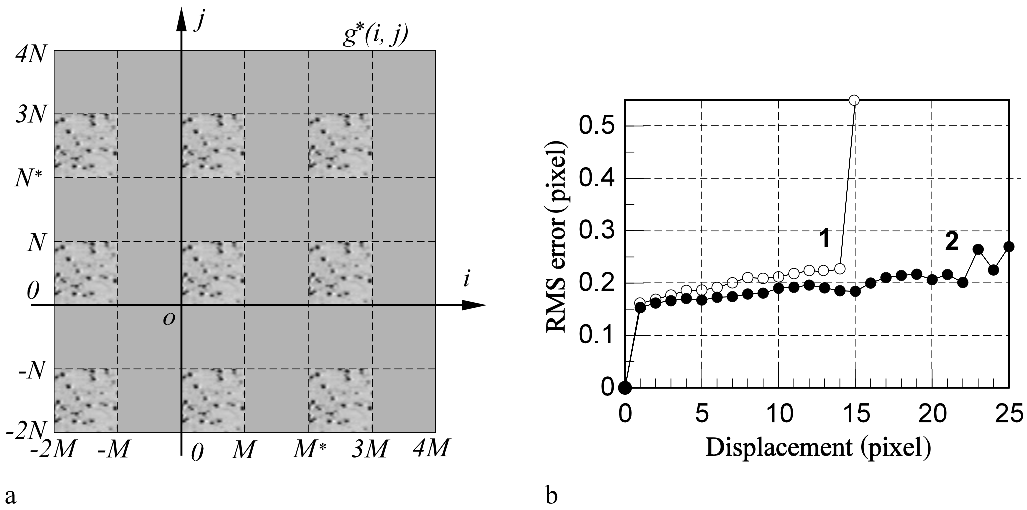

Fig. 3. **a** Periodical arrangement of interrogation area with "frame"; **b** RMS value of the error made in analyzing test cases with a displacement. *1* conventional evaluation; *2* interrogation window with "frame"

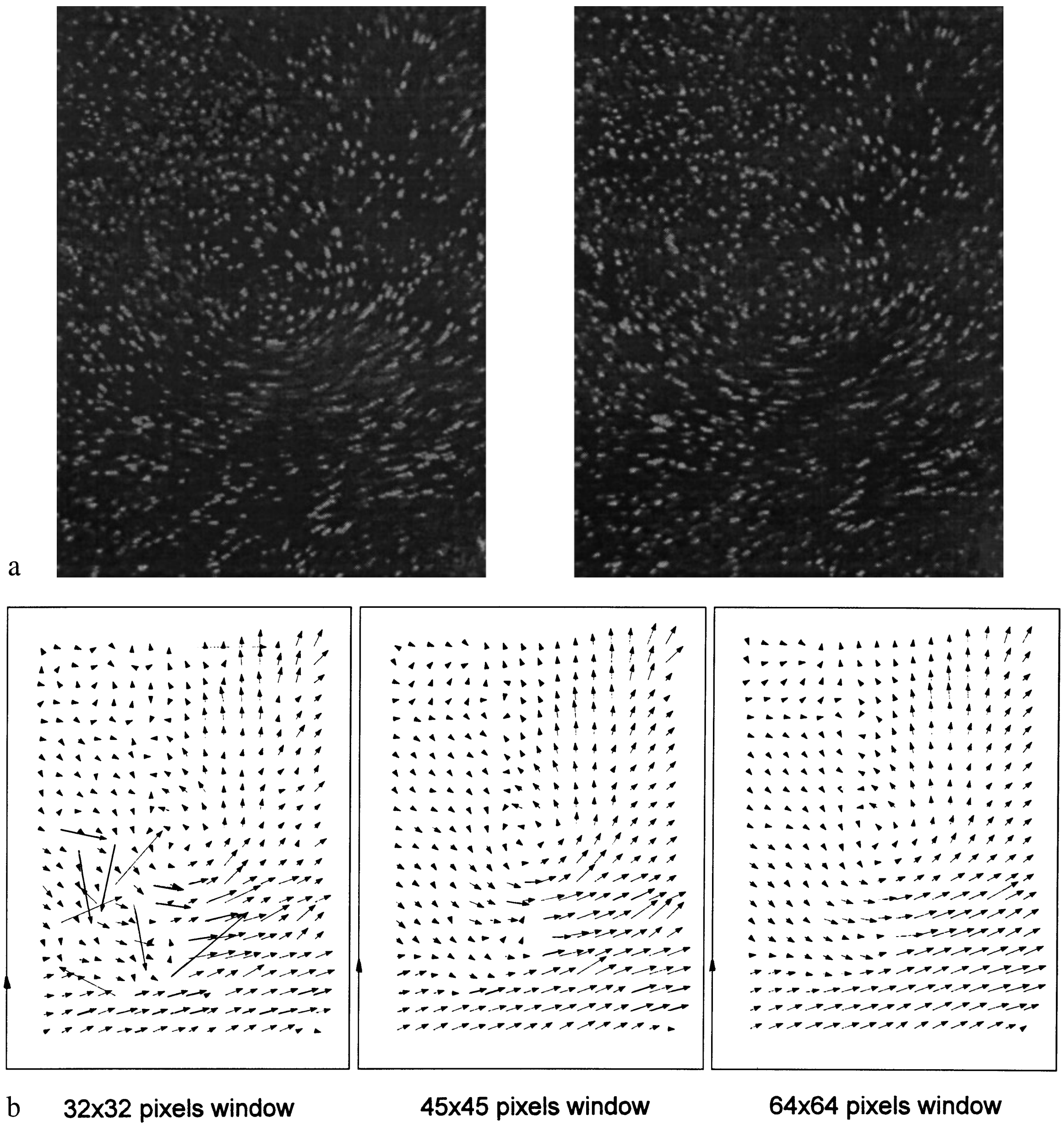

Fig. 4. **a** Two consecutive digital recordings of a water flow seeded with tracer particles. Size of recordings is 380 × 500 pixels. Time interval between recordings is 20 ms; **b** result of evaluation with differently sized interrogation windows

that the maximum displacement that can be measured for a given window size appears to be higher for the case with "frame", as discussed above.

The influence of the size of the interrogation window on the quality of an evaluation is shown with a second example. By means of cross-correlation we evaluate two successive PIV recordings taken in a water flow and covering an area of 7.6×10 cm (Fig. 4a). The digital recordings separated by a time interval of 20 ms have a format of 380×500 pixels. They are interrogated with three differently sized windows: 32×32 pixels, 45×45 pixels, 64×64 pixels (Fig. 4b). The window is shifted in steps of 20 pixels. The application of the smallest window results in a number of apparently wrong vectors, probably due to an insufficient number of particle images for the correlation. With the 64×64 pixels window one obtains a smooth vector field. This is the result of the high degree of averaging in this large window and the relatively large overlap (2/3 of the window size) in shifting the window. The smoothness in the distribution of the vectors is not a proof of the accuracy of the measurement. The loss of local resolution caused by the averaging may hide regimes of stronger velocity gradient, as they appear to occur in the lower right portion of the vector field. The result obtained with the 45×45 pixels window shows steeper velocity gradients in this part than it can be observed for the evaluation with the 64×64 pixels window, but with a considerable reduction of wrong vectors compared with the 32×32 pixels window.

In this example, the gain in evaluation speed, when applying the FFT to the 45×45 pixels window plus "frame", was about 20 as compared to evaluating the "unframed" 45×45 pixels window without FFT. But it should be noted that this gain in evaluation speed depends on the magnitude of the displacement. The evaluation accuracy of the technique described here and a direct correlation is the same.

4 Summary

The technique described here allows a higher flexibility in selecting the appropriate size of the interrogation window without losing the advantage of a rapid evaluation procedure by using the FFT algorithm. The local resolution of the measurement can be increased without simultaneously increasing the error caused by an insufficient number of particle images in the window. Also increased is the maximum displacement that can be determined for a chosen window size. The potential of the new technique is exemplified here with two test cases. A mathematical analysis would be required to prove the apparent technical advantages, but such an analysis would be beyond the scope of this Note.

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